Guiding Blind Transmitters: Degrees of Freedom Optimal Interference Alignment Using Relays

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Abstract—Channel state information (CSI) at the transmitters (CSIT) is of importance for interference alignment schemes to achieve the optimal degrees of freedom (DoF) for wireless networks. This paper investigates the impact of half-duplex relays on the degrees of freedom (DoF) of the X channel and the interference channel when the transmitters are blind in the sense that no CSIT is available. In particular, it is shown that adding relay nodes with global CSI to the communication model is sufficient to recover the DoF that is the optimal for these models with global CSI at the transmitters. The relay nodes in essence help steer the directions of the transmitted signals to facilitate interference alignment to achieve the optimal DoF with CSIT. The general $M \times N$ X channel with relays and the K-user interference channel are both investigated, and sufficient conditions on the number of antennas at the relays and the number of relays needed to achieve the optimal DoF with CSIT are established. Using relays, the optimal DoF can be achieved in finite channel uses. The DoF for the case when relays only have delayed CSI is also investigated, and it is shown that with delayed CSI at the relay the optimal DoF with full CSIT cannot be achieved. Special cases of the X channel and interference channel are investigated to obtain further design insights.

Index Terms—Degrees of freedom, interference alignment, relay, X channel, K-user interference channel, channel state information.

I. INTRODUCTION

Interference is inherent to any fully connected multi-user wireless network. As the number of devices sharing the spectrum with high rate demands grows, wireless networks become more and more interference limited. The significance of interference on the operation of a wireless network renders it natural to focus on its high SNR performance to obtain design insights and characterize the interaction between the signals. Thus, degrees of freedom (DoF), which characterizes the scaling of the transmission rates of wireless networks in high signal to noise ratio (SNR) regime, is an important metric to measure the performance of an interference-limited system.

Interference alignment was shown to achieve the optimal DoF for a variety of interference-limited wireless networks

[1]–[4]. In reference [1], the authors have shown that the optimal DoF $\frac{4M}{3}$ can be achieved for the 2-user multiple input multiple output (MIMO) X channel with M antennas at each node, using symbol extensions and interference alignment, demonstrating the achievability of non-integer DoF $\frac{4M}{3}$ with constant channel for $M > 1$. For $M = 1$ with constant channels, the DoF $\frac{4}{3}$ is shown to be achievable in [4]. Reference [2] further generalized the result to the $M \times N$ user X channel, and showed that the optimal DoF is $\frac{MN}{M+N-1}$ with single antenna nodes and a time-varying channel. Reference [3] showed that interference alignment achieves the optimal DoF of the K-user interference channel, $\frac{K}{2}$, with single antenna nodes and time-varying channel. Follow up studies on the DoF of the interference channels, for example, the SIMO interference channel, the K-user $M \times N$ MIMO interference channel, and interference channel with cooperation and cognition, can be found in references [5]–[7].

To effectively implement interference alignment, it is crucial to have global instant CSIT which can be difficult to obtain for practical systems. Reference [8] has studied the DoF region of the 2-user MIMO broadcast channel and the 2-user MIMO interference channel without CSIT, and loss of DoF is observed for many scenarios of interest. Reference [9] has further generalized the results to K-user broadcast and interference channels, and also derived outerbounds on DoF region for the K-user X channel. This reference has established that without CSIT, the transmitters cannot steer the signals to the exact desired directions to guarantee that the interference is aligned together at the receivers, which causes the performance degradation in terms of DoF.

While loss of DoF is observed when no CSIT is available at the transmitters, reference [10] has observed that as long as the channel's correlation structure is known at the transmitters, without any knowledge of the exact channel coefficient, interference alignment is still possible for certain wireless networks. Reference [11] has further developed this idea and proposed the blind interference alignment strategies using staggered antennas, which can artificially create the desired channel correlation pattern by switching the antennas used by the receivers. For systems where CSIT is completely unknown however, loss of DoF appears to be inevitable.

A more practical assumption about CSIT is that the transmitter may have delayed CSI. The delayed CSIT model characterizes the channel variation and the delay in the feedback of CSI from receivers, and thus is important from both theoretical and practical perspective. The delayed CSIT assumption is first studied in the context of the K-user broadcast channel [12], i.e., a channel with a transmitter having K antennas and K

receivers each with a single antenna, where the transmitter has accurate and global CSIT delayed by several time slots. It is shown that the delayed CSIT can be useful for interference alignment and the DoF can be improved significantly compared to the case without CSIT. This delayed CSIT assumption is then applied to various channel models such as the general broadcast channels, interference channels and X channels, and improvement on the DoF compared to the cases without CSIT can be found in references [13]–[19].

The delayed CSIT is an interesting assumption which, in fact, shows that feedback of delayed CSI can provide capacity gain for multi-destination wireless networks, which is in contrast with various single-destination models [20]–[22]. However, there is a performance degradation with the delayed CSIT assumption compared to when global CSIT is available. For example, the DoF for the K-user broadcast channel with delayed CSIT is shown in [12] to be $\frac{K}{1+\frac{1}{2}+\cdots+\frac{1}{K}}$, whereas with global CSIT, the optimal DoF is K.

The operation of relaying, although is beneficial in improving the achievable rates for many multi-user wireless networks [23]–[29], is shown in reference [30] to be unable to provide DoF gain for the fully connected interference channel and X channel with full CSI at all nodes. In this context, relaying is shown to be useful only to facilitate interference alignment for some specific scenarios. For quasi-static channels, references [31]–[33] have proposed strategies to utilize the relay to randomize the channel coefficients at the receivers, and the optimal DoF can be achieved although the channel is not time varying within the transmission blocks. Reference [34] has proposed relay-aided interference alignment schemes that can achieve the optimal DoF of the K-user interference channel with finite time extensions. For networks that have limited CSI, it is shown in reference [35] that using a relay, the optimal DoF for the K-user interference channel can be achieved when all the nodes have local CSI only, provided that the relay has more antennas than the total number of single-antenna transmitters. For networks that are not fully connected, for example, the multi-hop relay networks, references [18], [36]–[38] have studied the DoF under either global CSIT or delayed CSIT assumptions.

Whereas the study of relaying on the DoF of fully connected wireless networks so far focused on using relays to facilitate interference alignment, in this work, we aim to theoretically study the impact of relaying on the DoF from another perspective. We focus on understanding whether, and to what extent, relays can improve the DoF of wireless networks when the source nodes, i.e., the transmitters, are *blind* in the sense that no CSIT is available. In this paper, we mainly consider the case when relays have global CSI as a first step to investigate the impact of relays on the DoF of wireless networks without CSIT. The justification of the setting is that it is likely that the relay nodes are located in between the sources and the destinations and could have access to more accurate CSI. The relays can be small base stations at fixed locations with more power resources and computing capability, and obtaining CSI can be less challenging. Specifically, we study the DoF of the X channel and the interference channel with single-antenna users and half-duplex multi-antenna relays, where no

CSI is available at the transmitters, but global CSI can be obtained at the relays and the receivers. We first design a *joint beamforming* based transmission scheme for the general $M \times N$ X channel with relays. We show that when each relay is equipped with L antennas, with $\left\lceil \frac{(M-1)(N-1)}{L^2} \right\rceil$ relays, the DoF $\frac{MN}{M+N-1}$ can be achieved, which is the same optimal DoF as the case when CSI is available. We then consider two special cases: the K-user X channel with a multiple antenna relay and the K-user X channel with single antenna relays. For the case when relay nodes are only equipped with single antenna, the optimal DoF can be achieved with $(K-1)^2$ relays. For the case with one multiple antenna relay, we can design a different scheme with less computational complexity that uses *partial interference alignment* at the relay and joint beamforming to show the achievability of optimal DoF $\frac{K^2}{2K-1}$ using one relay with $K-1$ antennas. Note that in the above results, the channel is required to be time varying in order to achieve the optimal DoF. An interesting feature of the DoF optimal interference alignment scheme using relays is that only finite channel usage is required to achieve the exact optimal DoF, whereas for the general $M \times N$ X channel without relays but with CSIT, infinite channel uses are required. The case when there is no CSIT but relays only have delayed CSI is also investigated.

Using the techniques developed for the X channel, we further show that interference alignment is possible for the K-user interference channel without CSIT with the help of half-duplex relays. For the general case, we design a two-slot transmission scheme using joint beamforming, and show that it requires $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays with L antennas to achieve the DoF $\frac{K}{2}$, which is exactly the same optimal DoF as the case with CSIT. We then consider two special cases: the case with one relay with $K-1$ antennas, and the case with $K(K-2)$ single antenna relays. Note that the special case when the relay has $K-1$ antennas is also investigated in reference [39]. When we have one relay with $K-1$ antennas, joint beamforming is not necessary for interference alignment and the channel does not need to be time varying. However, when we have $K(K-2)$ relays each with a single antenna, joint beamforming is required to achieve interference alignment and the channel does need to be time varying.

Throughout the paper, we use bold letters, e.g. $\mathbf{h}$, to denote constant vectors, bold capital letters, e.g. $\mathbf{H}$, to denote matrices or vector of random variables, and ordinary capital letters, e.g. H, to denote random variables. We use $\lceil x \rceil$ to denote the closest integer that is smaller than x, and $\lfloor x \rfloor$ to denote the closest integer that is larger than x. $[a_i]^{(i)}$ denotes the column vector obtained by enumerating a_i with index i, i.e.,

$$[a_i]^{(i)} = [a_i]_{i=1}^{i=n} = [a_1, a_2, \cdots, a_n]^T, \qquad (1)$$

if $i = 1, 2, \cdots, n$.

Similarly, $[a_{ij}]^{(ij)}$ denotes the column vector

$$\left[\left[[a_{1j}]^{(j)} \right]^T, \left[[a_{2j}]^{(j)} \right]^T, \cdots \right]^T, \qquad (2)$$

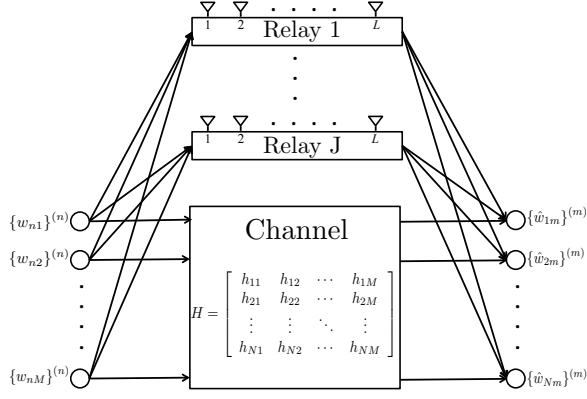which is obtained by enumerating a_{ij} for all indices i and j as its entries.

Fig. 1. $M \times N$ X channel with relays with $m = 1, \cdots, M$, $n = 1, \cdots, N$.

Fig. 2. K-user interference channel with relays.

We also use $[a_{ijk}]^{(ijk)}$ to denote the column vector

$$\left[\left[[a_{1jk}]^{(jk)} \right]^T, \left[[a_{2jk}]^{(jk)} \right]^T, \cdots \right]^T, \quad (3)$$

which is obtained by enumerating a_{ijk} for all indices i, j, k as its entries.

The remainder of the paper is organized as follows: Section II introduces the system model. Section III studies the relay-aided interference alignment schemes for the X channel. Section IV studies the relay-aided interference alignment schemes for the interference channel. Section V concludes the paper.

II. SYSTEM MODEL

A. $M \times N$ X channel with Relays

Fig. 1 shows the $M \times N$ X channel with relays. In this model, there are M transmitters and N receivers, and each transmitter has a message to be communicated to each receiver. It is assumed that the transmitters and receivers are equipped with single antenna. There are J half-duplex relays available to help the transmission. Each relay is assumed to have L antennas. We denote w_{nm} as the message intended from transmitter m to receiver n. The transmitted signal from transmitter m is denoted as $X_m(t) \in \mathbb{C}$ and the transmitted signal from relay R_j is denoted as $\mathbf{X}_{R_j}(t) \in \mathbb{C}^L$, where t is the time index denoting the slot in which the signal is transmitted.

When the relays listen to the channel, the received signals at the receivers are

$$Y_n(t) = \sum_{m=1}^{M} h_{nm}(t) X_m(t) + Z_n(t), \quad (4)$$

where $n = 1, \cdots N$, $m = 1, \cdots, M$, and the received signals at the relays are

$$\mathbf{Y}_{R_j}(t) = \sum_{m=1}^{M} \mathbf{h}_{R_j m}(t) X_m(t) + \mathbf{Z}_{R_j}(t), \ j = 1, \cdots, J. \quad (5)$$

When the relays transmit, the received signals at the receivers are

$$Y_n(t) = \sum_{m=1}^{M} h_{nm}(t) X_m(t) + \sum_{j=1}^{J} \mathbf{h}_{nR_j}(t)^T \mathbf{X}_{R_j}(t) + Z_n(t). \quad (6)$$
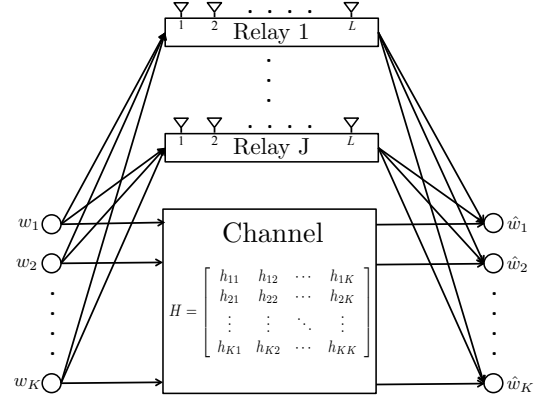
In the above expressions, the transmitted signals are subject to average power constraints $E(||\mathbf{X}_{R_j}(t)||^2) \leq P$, $E(|X_m(t)|^2) \leq P$, $j = 1, \cdots, J$, $m = 1, \cdots, M$. $h_{nm} \in \mathbb{C}$ is the channel coefficient from transmitter m to the receiver n. $\mathbf{h}_{R_j m}(t) \in \mathbb{C}^L$ is the channel vector between transmitter m and relay R_j, and $\mathbf{h}_{nR_j}(t) \in \mathbb{C}^L$ is the channel vector between relay R_j and receiver n. It is assumed that the channel coefficients are independently drawn from a continuous distribution for each time index, and the channel is time varying. $Z_n(t)$ and $\mathbf{Z}_{R_j}(t)$ are zero-mean Gaussian random variables with unit variance and identity covariance matrix, respectively.

We denote the rate of message w_{ij} with $R_{ij}(P)$ under power constraint P. Define $\mathcal{C}(P)$ as the set of all achievable rate tuples $[R_{nm}(P)]^{(nm)}$ under power constraint P. The DoF is defined as

$$DoF = \lim_{P \to \infty} \frac{R_{\sum}(P)}{\log(P)}, \quad (7)$$

where $R_{\sum}(P) = \max_{\mathcal{C}(P)} \left(\sum_{m,n} R_{nm}(P) \right)$. Note that since we consider the DoF as our metric, in the rest of the paper, we omit the noise terms in equations (4)-(6).

B. K-user Interference Channel with Relays

Fig. 2 shows the K-user interference channel with relays. In this model, there are K transmitters and K receivers, and each transmitter has a message to be communicated to one intended receiver. It is assumed that the transmitters and receivers are equipped with single antenna. There are J half-duplex relays available to help the transmission. Each relay is assumed to have L antennas. We denote w_k as the message intended from transmitter k to receiver k, $k = 1, \cdots, K$. The transmitted signal from transmitter k is denoted as $X_k(t) \in \mathbb{C}$ and the signal from relay j is denoted as $\mathbf{X}_{R_j}(t) \in \mathbb{C}^L$, where t is the time index denoting the slot in which the signal is transmitted.

When the relays listen to the channel, the received signals at the receivers are

$$Y_n(t) = \sum_{k=1}^{K} h_{nk}(t) X_k(t) + Z_n(t), \ n = 1, \cdots, K \quad (8)$$

and the received signals at the relays are

$$\mathbf{Y}_{R_j}(t) = \sum_{k=1}^{K} \mathbf{h}_{R_j k}(t) X_k(t) + \mathbf{Z}_{R_j}(t), \ J = 1, \cdots, J. \quad (9)$$

When relays transmit, the received signals at the receivers are

$$Y_n(t) = \sum_{k=1}^{K} h_{nk}(t)X_k(t) + \sum_{j=1}^{J} \mathbf{h}_{nR_j}(t)^T \mathbf{X}_{R_j}(t) + Z_n(t). \tag{10}$$

The power constraints on the transmitted signals, the channel coefficients, and the channel noise are defined as in Section II-A.

We denote the rate of message w_k is $R_k(P)$ under power constraint P. Define $\mathcal{C}(P)$ as the set of all achievable rate tuples $[R_k(P)]^{(k)}$ under power constraint P. The DoF is defined as in (7) with the sum rate now defined as $R_{\sum}(P) = \max_{\mathcal{C}(P)} \left(\sum_{k=1}^{K} R_k(P) \right)$. We ignore the noise terms in equations (8)-(10) in the sequel.

III. RELAY-AIDED INTERFERENCE ALIGNMENT FOR X CHANNEL WITHOUT CSIT

In this section, we provide the DoF for the $M \times N$ X channel, with the assumption that the transmitters have no CSI, and relay nodes with global CSI are present to help. Without CSIT, the transmitters cannot send the signals in the desired directions to align the interference at the receivers. However, as we shall show next, relays can be used to help the transmitters steer the directions of the transmitted signals to achieve the DoF as if global CSI were available at the transmitters.

Before presenting the relay-aided interference alignment schemes, we first find an upper bound for the DoF of the $M \times N$ X channel without CSIT, but with relays.

Proposition 1: For the $M \times N$ X channel without CSIT, where relays have global CSI, the DoF is upper bounded by $\frac{MN}{M+N-1}$, regardless of the number of relays and the number of antennas at the relays.

Proof: The $M \times N$ X channel without CSIT with relays can be upper bounded by the $M \times N$ X channel with CSIT and relays. Note that here we consider arbitrary number of relays with arbitrary number of antennas. Reference [2] showed that with global CSI at all nodes, the optimal DoF of the $M \times N$ X channel is $\frac{MN}{M+N-1}$. Reference [30] further showed that relaying does not increase the DoF of X channels, when all nodes are equipped with global CSI. This means that the $(M \times N)$-user X channel with CSIT and relays with global CSI has optimal DoF $\frac{MN}{M+N-1}$, which is clearly an upper bound for the $M \times N$ X channel without CSIT with relays. ∎

Remark 1: Note that since there is no assumption about whether the channel is time varying or not in the arguments for outerbounds on DoF in references [2], [30], the DoF upper bound we have in *Proposition 1* is valid for both time varying and constant channels.

Now, we can proceed to construct the relay-aided interference alignment schemes to show that, with the help of relays, the DoF upperbound $\frac{MN}{M+N-1}$, which is obtained by assuming global CSIT, is in fact achievable without CSIT. Observe that for the K-user X channel, the DoF upperbound reduces to $\frac{K^2}{2K-1}$.

A. $M \times N$ X Channel with J Relays with L antennas

We first consider the $M \times N$ X channel with J relays each having L antennas and design transmission schemes that can achieve the DoF upper bound in *Proposition 1* without using CSIT.

Theorem 1: For the $M \times N$ X channel with J relays each having L antennas, when the transmitters have no CSIT but the relays have global CSI, a sufficient condition to achieve the optimal DoF $\frac{MN}{M+N-1}$ is that $J \geq \left\lceil \frac{(M-1)(N-1)}{L^2} \right\rceil$.

Proof: For the $M \times N$ X channel, each transmitter has a message for each receiver, and we wish to deliver the MN messages to the desired receivers in $M + N - 1$ slots.

We label the relays with R_i where $i = 1, 2, \cdots, J$. For slots $t = 1, 2, \cdots, N$, the transmitters send the messages to the receivers, and the relays remain silent. Specifically, the signal sent from transmitter m at slot t is

$$X_m(t) = d_{tm}, \tag{11}$$

where d_{tm} is the data stream carrying the message W_{tm}.

The received signals at receiver n and relay R_i are

$$Y_n(t) = \sum_{m=1}^{M} h_{nm}(t)d_{tm} \tag{12}$$

$$\mathbf{Y}_{R_i}(t) = \sum_{m=1}^{M} \mathbf{h}_{R_i m}(t)d_{tm}. \tag{13}$$

where $\mathbf{Y}_{R_i}(t) \in \mathbb{C}^L$.

For slots $t' = N + 1, \cdots, M + N - 1$, each relay R_i constructs a precoding matrix $\mathbf{U}_{it}(t') \in \mathbb{C}^{L \times L}$ for the signals received in each previous slot t, and transmits the following signal in slot t':

$$X_{R_i}(t') = \sum_{t=1}^{N} \mathbf{U}_{it}(t')\mathbf{Y}_{R_i}(t) \tag{14}$$

In addition, for slot t', transmitter 1 also sends the following signal to the receivers:

$$X_1(t') = \sum_{n=1}^{N} d_{n1}. \tag{15}$$

The signal received at receiver n for slot t' is thus

$$Y_n(t') = \sum_{j=1}^{N} h_{n1}(t')d_{j1}$$
$$+ \sum_{i=1}^{J} \sum_{t=1}^{N} \sum_{m=1}^{M} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{it}(t')\mathbf{h}_{R_i m}(t)d_{tm}. \tag{16}$$

After combining all the received signals from every slot, the resulting signal can be expressed as in equation (17) at the beginning of next page. Note that in equation (17), n, t' and γ outside the parenthesis of the vectors denote the nth, t'th and γth entry of the vectors, and we have utilized the notation defined in equations (1)-(3).

$$\mathbf{Y}_n = \begin{array}{c} n \\ \\ t' \end{array} \left[\begin{array}{c} \mathbf{0} \\ h_{n1}(n) \\ \mathbf{0} \\ \left[h_{n1}(t') + \sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{in}(t') \mathbf{h}_{R_i1}(n) \right]^{(t')} \end{array} \right] d_{n1} + \sum_{k \neq 1} \begin{array}{c} \\ \\ \\ \end{array} \left[\begin{array}{c} \mathbf{0} \\ h_{nk}(n) \\ \mathbf{0} \\ \left[\sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{in}(t') \mathbf{h}_{R_ik}(n) \right]^{(t')} \end{array} \right] d_{nk}$$

$$+ \sum_{\gamma \neq n} \left(\begin{array}{c} \gamma \\ \\ t' \end{array} \left[\begin{array}{c} \mathbf{0} \\ h_{n1}(\gamma) \\ \mathbf{0} \\ \left[h_{n1}(t') + \sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{i\gamma}(t') \mathbf{h}_{R_i1}(\gamma) \right]^{(t')} \end{array} \right] d_{\gamma 1} \right.$$

$$\left. + \sum_{k \neq 1} \begin{array}{c} \gamma \\ \\ t' \end{array} \left[\begin{array}{c} \mathbf{0} \\ h_{nk}(\gamma) \\ \mathbf{0} \\ \left[\sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{i\gamma}(t') \mathbf{h}_{R_ik}(\gamma) \right]^{(t')} \end{array} \right] d_{\gamma k} \right) \tag{17}$$

In order to align all the interference messages into an $N-1$ dimensional space, we choose the precoding matrices $\mathbf{U}_{it}(t')$ such that

$$\frac{\sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{i\gamma}(t') \mathbf{h}_{R_ik}(\gamma)}{h_{nk}(\gamma)}$$
$$= \frac{h_{n1}(t') + \sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{i\gamma}(t') \mathbf{h}_{R_i1}(\gamma)}{h_{n1}(\gamma)}, \tag{18}$$

which can be written as

$$\sum_{i=1}^{J} \mathbf{h}_{nR_i}(t')^T \mathbf{U}_{i\gamma}(t') \left(h_{n1}(\gamma) \mathbf{h}_{R_ik}(\gamma) - h_{nk}(\gamma) \mathbf{h}_{R_i1}(\gamma) \right)$$
$$= h_{nk}(\gamma) h_{n1}(t'), \tag{19}$$

for all $t' = N+1, \cdots, M+N-1$.

If we denote the entry for pth row and qth column of matrix $\mathbf{U}_{i\gamma}(t')$ as $u_{i\gamma,p,q}(t')$, where $p,q = 1, \cdots, L$, we can define a vector

$$\mathbf{u}(\gamma, t') = [u_{i\gamma,p,q}(t')]^{(ipq)}, \tag{20}$$

where the notation $[u_{i\gamma,p,q}(t')]^{(ipq)}$ is defined as in equation (1)-(3). We also define vectors

$$\mathbf{h}_{n,k}(\gamma, t') = [h_{nR_i,p}(t') (h_{n1}(\gamma) h_{R_ik,q}(\gamma)$$
$$- h_{nk}(\gamma) h_{R_i1,q}(\gamma))]^{(ipq)}, \tag{21}$$

and matrix $\mathbf{H}(\gamma, t')$, which is formed by taking $\mathbf{h}_{n,k}(\gamma, t')^T$ as its rows for all enumeration of n and k.

All the linear equations can now be written as

$$\mathbf{H}(\gamma, t') \mathbf{u}(\gamma, t') = \mathbf{b}(\gamma, t'), \tag{22}$$

where $\mathbf{b}(\gamma, t') = [h_{nk}(\gamma) h_{n1}(t')]^{(nk)}$.

Since we have one equation for each pair of (n,k) where $k \neq 1, n \neq \gamma$, there are $(M-1)(N-1)$ equations for each pair of fixed (γ, t'). On the other hand, each matrix $\mathbf{U}_{i\gamma}(t')$ can provide L^2 variables, which gives us JL^2 variables in total. When $J \geq \left\lceil \frac{(M-1)(N-1)}{L^2} \right\rceil$, we can guarantee that there exist solutions to the equations to find the matrices $\mathbf{U}_{i\gamma}(t')$.

In the sequel, we drop the parameters (γ, t') in the expression for matrix $\mathbf{H}(\gamma, t')$ for clarity. Since the channel coefficients are drawn from a continuous distribution, the matrix $\mathbf{H}$ is of full rank almost surely. When the matrix $\mathbf{H}$ is square, the relays can find the precoding matrix by calculating $\mathbf{H}^{-1}\mathbf{b}$. When the matrix $\mathbf{H}$ is not square, since $J \geq \left\lceil \frac{(M-1)(N-1)}{L^2} \right\rceil$, the vector $\mathbf{u}$ can be calculated using $\mathbf{H}^{\dagger}(\mathbf{H}\mathbf{H}^{\dagger})^{-1}\mathbf{b}$. The calculation of the precoding matrices for both cases only requires global CSI at the relays and no cooperation between the relays is needed.

With the matrices $\mathbf{U}_{i\gamma}(t')$, all the interfering signals can be aligned into an $N-1$ dimensional space. We now need to verify that the interference and the signals carrying intended messages are linearly independent. Since for receiver n, the signals carrying intended messages and the interfering signals do not have non-zero entries in the same row of the received signal vector from row 1 to row N, as shown in equation (17), it is guaranteed that the signals are linearly independent. As an example, consider the channel with $M = N = 3$ and receiver 1. The received signal is of the form

$$\mathbf{Y}_1 = \begin{bmatrix} a \\ 0 \\ 0 \\ b \\ c \end{bmatrix} d_{intended} + \begin{bmatrix} 0 \\ e \\ 0 \\ g \\ h \end{bmatrix} d_{interference}. \tag{23}$$

It can be readily seen that the signal vectors carrying intended messages and the ones with interference in equation (23) are linearly independent.

This special structure of the received signals is originated from the design of the transmission scheme. The fact that the channel coefficients are drawn from a continuous distribution guarantee that the desired data streams occupy the rest M dimensional space, and thus a zero-forcing decoder can recover all the desired messages to achieve DoF $\frac{MN}{M+N-1}$. ∎

Remark 2: Note that in the scheme, when $JL^2 = (M-1)(N-1)$, joint beamforming is mandatory to obtain the precoding matrices $\mathbf{U}_{i\gamma}(t')$. This is because when $JL^2 = (M-1)(N-1)$, the matrix $\mathbf{H}(\gamma, t')$ in equation (22) is invertible, and the vector $\mathbf{b}(\gamma, t')$ becomes zero if joint beamforming is not utilized. The precoding matrices at the relays thus are all zero. In this way, the interference can still be aligned since

they occupy different time indices than the intended signals, but there is not sufficient dimension to decode the intended messages. Joint beamforming, for this case, can guide the relays to steer the interference such that they are aligned, and in the meantime guarantee that there is sufficient signal dimension. On the other hand, when $JL^2 > (M-1)(N-1)$, joint beamforming is not required, since we can always find a non-zero vector from the null space of matrix $\mathbf{H}(\gamma, t')$. ∎

Remark 3: In the transmission scheme, the time varying nature of the channel is crucial for the receivers to decode the intended signals. From equation (17), we can see that when channel is not time varying, the intended signals fall into a space of dimension 2, and the receivers cannot decode all the intended messages. ∎

Remark 4: For the X channel without CSIT and without relays, reference [9] has shown that the DoF upperbound is 1 when the channel experiences Rayleigh fading, provided that all nodes are equipped with a single antenna. Our result shows that relaying is useful to provide DoF gain for the X channel without CSIT. This is to be contrasted with the result in reference [30], which has shown that relaying cannot provide DoF gain for the X channel when global CSI is available at all the nodes. ∎

Remark 5: We have shown that using relays, we can achieve the optimal DoF for the $M \times N$ X channel in finite channel uses. However, for the case with global CSIT but no relays are available, the same optimal DoF is achievable using infinite channel uses, as shown in reference [2]. ∎

Remark 6: The scheme we used in *Theorem 1* can be generalized to the case when each user has multiple antennas by counting the number of equations required for interference alignment and the number of variables that can be provided by the relays. ∎

We next investigate a special case of the general X channel, which is the K-user X channel with a single relay equipped with multiple antennas. For this case, we can design a different scheme using the available spatial dimension at the relay, which can provide more insights regarding how interference signals are aligned and has lower computational complexity for the relay to obtain the precoding matrices.

B. The K-user X Channel with one Multi-antenna Relay

For the K-user X channel without CSIT, when we have a relay with K antennas, the relay can decode all the data streams sent from the transmitters, for example with zero-forcing, if each transmitter only sends a single data stream with DoF 1. Since the relay has global CSI, clearly it can perform appropriate precoding to align the interfering signals at the receivers. The result from *Theorem 1* implies that for this case, $K-1$ antennas are in fact sufficient for the relay to align the interference at the receivers to achieve the DoF upperbound $\frac{K^2}{2K-1}$.

When the relay has multiple antennas, we can use a different strategy than the one we used to prove *Theorem 1* to achieve the DoF upper bound. To better illustrate the transmission strategy, we first provide an example for the 3-user X channel with a relay having 2 antennas, and then generalize the

scheme to the K-user case. Note that when $K = 3$, the DoF upperbound becomes $\frac{9}{5}$.

1) 3-user X Channel with a Relay with 2 Antennas:

Corollary 1: For the 3-user X channel without CSIT with relays, optimal DoF $\frac{9}{5}$ is achievable using a relay with 2 antennas and global CSI.

Proof: We denote the data stream from transmitter i to receiver j as d_{ji}, $i, j = 1, 2, 3$, and data stream d_{ji} carries a message w_{ji}. To achieve the DoF $\frac{9}{5}$, we let each transmitter send one message to each receiver in 5 time slots. Note that the channel is assumed to be time varying for each slot, and the channel coefficients are drawn from a continuous distribution.

In the first 3 slots, the transmitters send messages to the receivers, while the relay keeps silent. Specifically, in slot t, all the 3 transmitters send the messages intended for receiver t:

$$X_k(t) = d_{tk} \tag{24}$$

where $t, k = 1, 2, 3$. At slot t, the received signals at the receivers and the relay are

$$Y_1(t) = h_{11}(t)d_{t1} + h_{12}(t)d_{t2} + h_{13}(t)d_{t3} \tag{25}$$

$$Y_2(t) = h_{21}(t)d_{t1} + h_{22}(t)d_{t2} + h_{23}(t)d_{t3} \tag{26}$$

$$Y_3(t) = h_{31}(t)d_{t1} + h_{32}(t)d_{t2} + h_{33}(t)d_{t3} \tag{27}$$

$$\mathbf{Y}_R(t) = \mathbf{h}_{R1}(t)d_{t1} + \mathbf{h}_{R2}(d)_{t2} + \mathbf{h}_{R3}(t)d_{t3}, \tag{28}$$

where we discarded the channel noise since we are considering the DoF of the channel.

In the remaining 2 slots, the relay needs to provide each receiver with two more equations such that the intended messages, which are the unknown variables d_{ji} in the equations, can be recovered. In the meantime, all the interference data streams must be kept in a 2-dimensional space at each receiver to achieve the optimal DoF. Since the relay has 2 antennas, it cannot decode all the three messages from each user to perform appropriate precoding in the remaining 2 slots. However, as we shall see, the spatial dimensions available at the relay can still be utilized to align the interference.

The relay first performs a linear transformation to the received signals using vectors $\mathbf{u}_i(t) \in \mathbb{C}^2$, where $i, t = 1, 2, 3$, $i \neq t$. Specifically, for $t = 1$, we want to partially align the interference caused by the messages intended for receiver 1. We design the vectors $\mathbf{u}_2(1)$ and $\mathbf{u}_3(1)$ such that they satisfy

$$\mathbf{u}_2(1)^T\mathbf{h}_{R2}(1) = h_{22}(1) \qquad \mathbf{u}_2(1)^T\mathbf{h}_{R3}(1) = h_{23}(1), \tag{29}$$

$$\mathbf{u}_3(1)^T\mathbf{h}_{R2}(1) = h_{32}(1) \qquad \mathbf{u}_3(1)^T\mathbf{h}_{R3}(1) = h_{33}(1). \tag{30}$$

Since we have two variables with two equations for each vector $\mathbf{u}_i(1)$ and the channel coefficients are drawn from a continuous distribution, we can guarantee the existence of $\mathbf{u}_2(1)$ and $\mathbf{u}_3(1)$ almost surely. We can then obtain the following signals by taking the inner products between the vector $\mathbf{u}_2(1)$ ($\mathbf{u}_3(1)$) and the received signal vector from slot 1:

$$\mathbf{u}_2(1)^T\mathbf{Y}_R(1) = \mathbf{u}_2(1)^T\mathbf{h}_{R1}(1)d_{11} + h_{22}(1)d_{12} + h_{23}(1)d_{13} \tag{31}$$

$$\mathbf{u}_3(1)^T\mathbf{Y}_R(1) = \mathbf{u}_3(1)^T\mathbf{h}_{R1}(1)d_{11} + h_{32}(1)d_{12} + h_{33}(1)d_{13}. \tag{32}$$

These two signals are useful for receiver 1, since they contain the messages that are intended for it. However, the messages d_{11}, d_{12} and d_{13} are interference for receiver 2 and 3. Using the linear transformation provided by vector $\mathbf{u}_2(1)$ or $\mathbf{u}_3(1)$, we can see that the channel coefficients for d_{12} and d_{13} in equation (31) are the same as the signal received at receiver 2. Similarly, the channel coefficients for d_{12} and d_{13} in equation (32) are the same as the signal received at receiver 3. If we can keep $\mathbf{u}_2(1)^T\mathbf{Y}_R(1)$ away from receiver 3, and keep $\mathbf{u}_3(1)^T\mathbf{Y}_R(1)$ away from receiver 2, part of the interference is aligned at receiver 2 and receiver 3. This can be done by sending $\mathbf{u}_2(1)^T\mathbf{Y}_R(1)$ and $\mathbf{u}_3(1)^T\mathbf{Y}_R(1)$ along the directions

$$\mathbf{v}_{12}(t) \perp \mathbf{h}_{3R}(t) \quad \text{and} \quad \mathbf{v}_{13}(t) \perp \mathbf{h}_{2R}(t), \qquad (33)$$

respectively, where $t = 4, 5$.

For the interference caused by the messages for receiver 2 and receiver 3, we design the precoding vectors $\mathbf{u}_1(2)$, $\mathbf{u}_3(2)$, $\mathbf{u}_1(3)$, and $\mathbf{u}_2(3)$ in the same fashion as we design the vectors $\mathbf{u}_2(1)$, $\mathbf{u}_3(1)$, which have the following properties:

$$\mathbf{u}_1(2)^T\mathbf{Y}_R(2) = \mathbf{u}_1(2)^T\mathbf{h}_{R2}(2)d_{22} + h_{11}(2)d_{21} + h_{13}(2)d_{23} \tag{34}$$

$$\mathbf{u}_3(2)^T\mathbf{Y}_R(2) = \mathbf{u}_3(2)^T\mathbf{h}_{R2}(2)d_{22} + h_{31}(2)d_{21} + h_{33}(2)d_{23} \tag{35}$$

$$\mathbf{u}_1(3)^T\mathbf{Y}_R(3) = \mathbf{u}_1(3)^T\mathbf{h}_{R3}(3)d_{33} + h_{11}(3)d_{31} + h_{12}(3)d_{32} \tag{36}$$

$$\mathbf{u}_2(3)^T\mathbf{Y}_R(3) = \mathbf{u}_2(3)^T\mathbf{h}_{R3}(3)d_{33} + h_{21}(3)d_{31} + h_{22}(3)d_{32}. \tag{37}$$

In order to transmit the signals along their intended directions, we now define the following beamforming vectors, which is similar as the vectors $\mathbf{v}_{12}(t)$ and $\mathbf{v}_{13}(t)$:

$$\mathbf{v}_{21}(t) \perp \mathbf{h}_{3R}(t) \quad \mathbf{v}_{23}(t) \perp \mathbf{h}_{1R}(t) \tag{38}$$

$$\mathbf{v}_{31}(t) \perp \mathbf{h}_{2R}(t) \quad \mathbf{v}_{32}(t) \perp \mathbf{h}_{1R}(t) \tag{39}$$

where $t = 4, 5$. We can choose the vectors such that they have unit power, and satisfy

$$\mathbf{v}_{31}(t) = \mathbf{v}_{13}(t) = \mathbf{v}_2^\perp(t), \tag{40}$$

$$\mathbf{v}_{12}(t) = \mathbf{v}_{21}(t) = \mathbf{v}_3^\perp(t), \tag{41}$$

$$\mathbf{v}_{23}(t) = \mathbf{v}_{32}(t) = \mathbf{v}_1^\perp(t). \tag{42}$$

Using the linear transformation and beamforming provided above, interference is only partially aligned. To align the rest of the interference, we let the relay choose a scaling factor $\alpha_{ij}(t)$ for each signal it wishes to send to the receivers, and produce the signals to be transmitted for slot 4 and slot 5 as shown in equation (43) at the beginning of next page, where the scalars $\alpha_{ij}(t)$ are to be determined later.

For slots 4 and 5, the transmitters also send the following signals to the receivers:

$$X_k(t) = d_{kk} \tag{44}$$

where $k = 1, 2, 3$. Note that other combinations of transmitted messages also work for our scheme.

The received signals at the receivers can be expressed as

$$Y_1(t) = h_{11}(t)d_{11} + h_{12}(t)d_{22} + h_{13}(t)d_{33} + \mathbf{h}_{1R}(t)^T\mathbf{X}_R(t) \tag{45}$$

$$Y_2(t) = h_{21}(t)d_{11} + h_{22}(t)d_{22} + h_{23}(t)d_{33} + \mathbf{h}_{2R}(t)^T\mathbf{X}_R(t) \tag{46}$$

$$Y_3(t) = h_{31}(t)d_{11} + h_{32}(t)d_{22} + h_{33}(t)d_{33} + \mathbf{h}_{3R}(t)^T\mathbf{X}_R(t) \tag{47}$$

If we combine all the received signals from 5 slots into a vector in $\mathbb{C}^5$, the resulting signal is shown in equation (48) at the beginning of next page. where we denote $h_{kR}^{\perp i}(t) = \mathbf{h}_{kR}(t)^T\mathbf{v}_i^\perp(t)$, $\mu_k^{Ri}(t) = \mathbf{u}_k(t)^T\mathbf{h}_{Ri}(t)$.

From the above expression, we can see that the data streams d_{21} and d_{23} are aligned in a one-dimensional space, and the data streams d_{31} and d_{32} are aligned in a one dimensional space. To align the data stream d_{22} with d_{21} and d_{23}, we choose

$$\frac{h_{12}(t) + \alpha_{21}(t)h_{1R}^{\perp 3}(t)\mu_1^{R2}(2)}{h_{12}(2)} = \alpha_{21}(t)h_{1R}^{\perp 3}(t), \tag{49}$$

which is equivalent as

$$\alpha_{21}(t) = \frac{h_{12}(t)}{(h_{12}(2) - \mu_1^{R2}(2))h_{1R}^{\perp 3}(t)}, \tag{50}$$

where $t = 4, 5$.

Similarly, to align d_{33} with d_{31} and d_{32} we choose

$$\alpha_{31}(t) = \frac{h_{13}(t)}{(h_{13}(3) - \mu_1^{R3}(3))h_{1R}^{\perp 2}(t)}. \tag{51}$$

The remaining parameters $\alpha_{12}(t), \alpha_{32}(t), \alpha_{13}(t), \alpha_{23}(t)$ can be determined in a similar fashion. It is easy to verify that the data streams d_{11}, d_{12} and d_{13} still occupy a 3-dimension space with the specified parameters $\alpha_{ij}(t)$. This argument holds at receiver 2 and receiver 3 as well. Hence using the proposed scheme, we can transmit a total of 9 messages using 5 slots, which proves the achievability of DoF $\frac{9}{5}$. ∎

Remark 7: We can see from equations (50) and (51) that joint beamforming is a key step to achieve the DoF upper bound. This is because without joint beamforming, i.e., transmitters stay silent for slot 4 and slot 5, the channel coefficients $h_{ij}(4)$ and $h_{ij}(5)$ are all zero. As a result, all the parameters $\alpha_{ij}(t)$ become zero. Similar as *Remark 2*, without joint beamforming, the interference signals can still be aligned, but there is not sufficient dimension for the receivers to decode the intended signals. ∎

Remark 8: From equation (48), we can see that the channel needs to be time varying for the receivers to have sufficient dimension to decode the intended signals, following similar arguments as in *Remark 3*. ∎

The idea of the above transmission strategy is to use the limited spatial dimensions available at the relay to first *partially* align the interference, and then align the rest of the interference through joint beamforming with the transmitters. Without the relay, the transmitters cannot send the signals at the intended directions for interference alignment since there is no CSIT, and reference [9] has shown that the DoF of the X channel for this case collapses to 1. The advantage of having the relays to assist interference alignment is thus obvious.

$$\mathbf{X}_R(t) = \alpha_{12}(t)\mathbf{v}_3^{\perp}(t)\left(\mathbf{u}_2(1)^T\mathbf{Y}_R(1)\right) + \alpha_{13}(t)\mathbf{v}_2^{\perp}(t)\left(\mathbf{u}_3(1)^T\mathbf{Y}_R(1)\right) + \alpha_{21}(t)\mathbf{v}_3^{\perp}(t)\left(\mathbf{u}_1(2)^T\mathbf{Y}_R(2)\right)$$
$$+ \alpha_{23}(t)\mathbf{v}_1^{\perp}(t)\left(\mathbf{u}_3(2)^T\mathbf{Y}_R(2)\right) + \alpha_{31}(t)\mathbf{v}_2^{\perp}(t)\left(\mathbf{u}_1(3)^T\mathbf{Y}_R(3)\right) + \alpha_{32}(t)\mathbf{v}_1^{\perp}(t)\left(\mathbf{u}_2(3)^T\mathbf{Y}_R(3)\right) \tag{43}$$

$$\mathbf{Y}_1 = \begin{bmatrix} h_{11}(1) \\ 0 \\ 0 \\ h_{11}(4) + \alpha_{12}(4)h_{1R}^{\perp 3}(4)\mu_2^{R1}(1) + \alpha_{13}(4)h_{1R}^{\perp 2}(4)\mu_3^{R1}(1) \\ h_{11}(5) + \alpha_{12}(5)h_{1R}^{\perp 3}(5)\mu_2^{R1}(1) + \alpha_{13}(5)h_{1R}^{\perp 2}(5)\mu_3^{R1}(1) \end{bmatrix} d_{11} + \begin{bmatrix} h_{12}(1) \\ 0 \\ 0 \\ \alpha_{12}(4)h_{1R}^{\perp 3}(4)h_{22}(1) + \alpha_{13}(4)h_{1R}^{\perp 2}(4)h_{32}(1) \\ \alpha_{12}(5)h_{1R}^{\perp 3}(5)h_{22}(1) + \alpha_{13}(5)h_{1R}^{\perp 2}(5)h_{32}(1) \end{bmatrix} d_{12}$$

$$+ \begin{bmatrix} h_{13}(1) \\ 0 \\ 0 \\ \alpha_{12}(4)h_{1R}^{\perp 3}(4)h_{23}(1) + \alpha_{13}(4)h_{1R}^{\perp 2}(4)h_{33}(1) \\ \alpha_{12}(5)h_{1R}^{\perp 3}(5)h_{23}(1) + \alpha_{13}(5)h_{1R}^{\perp 2}(5)h_{33}(1) \end{bmatrix} d_{13} + \begin{bmatrix} 0 \\ h_{11}(2) \\ 0 \\ \alpha_{21}(4)h_{1R}^{\perp 3}(4)h_{11}(2) \\ \alpha_{21}(5)h_{1R}^{\perp 3}(5)h_{11}(2) \end{bmatrix} d_{21} + \begin{bmatrix} 0 \\ h_{13}(2) \\ 0 \\ \alpha_{21}(4)h_{1R}^{\perp 3}(4)h_{13}(2) \\ \alpha_{21}(5)h_{1R}^{\perp 3}(5)h_{13}(2) \end{bmatrix} d_{23}$$

$$+ \begin{bmatrix} 0 \\ h_{12}(2) \\ 0 \\ h_{12}(4) + \alpha_{21}(4)h_{1R}^{\perp 3}(4)\mu_1^{R2}(2) \\ h_{12}(5) + \alpha_{21}(5)h_{1R}^{\perp 3}(5)\mu_1^{R2}(2) \end{bmatrix} d_{22} + \begin{bmatrix} 0 \\ 0 \\ h_{11}(3) \\ \alpha_{31}(4)h_{1R}^{\perp 2}(4)h_{11}(3) \\ \alpha_{31}(5)h_{1R}^{\perp 2}(5)h_{11}(3) \end{bmatrix} d_{31}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ h_{12}(3) \\ \alpha_{31}(4)h_{1R}^{\perp 2}(4)h_{12}(3) \\ \alpha_{31}(5)h_{1R}^{\perp 2}(5)h_{12}(3) \end{bmatrix} d_{32} + \begin{bmatrix} 0 \\ 0 \\ h_{13}(3) \\ h_{13}(4) + \alpha_{31}(4)h_{1R}^{\perp 2}(4)\mu_1^{R3}(3) \\ h_{13}(5) + \alpha_{31}(5)h_{1R}^{\perp 2}(5)\mu_1^{R3}(3) \end{bmatrix} d_{33} \tag{48}$$

Using the ideas from the example for the 3-user X channel with a 2-antenna relay, we can now generalize the result to the K-user case.

2) K-user X Channel with one $(K-1)$-antenna Relay:

Corollary 2: For the K-user X channel without CSIT with relays, the optimal DoF $\frac{K^2}{2K-1}$ is achievable using one relay with $K-1$ antennas and global CSI.

Proof: The achievability of DoF $\frac{K^2}{2K-1}$ follows the idea from *Corollary 1*, and the detailed scheme is provided in Appendix A. ∎

Remark 9: The schemes provided in *Corollary 1* and *Corollary 2* can be seen as specific construction of the precoding matrices at the relay, where partial interference alignment and joint beamforming are utilized. The scheme we used in *Corollary 1* and *Corollary 2* has more of a straight forward physical interpretation, and more importantly, it has lower computational complexity since it only requires $K \times K$ matrix inversion when finding the vectors $\mathbf{u}_i(t)$ and $\mathbf{v}_{ti}(t')$. In comparison, the scheme we used in *Theorem 1* requires matrix inversion operation of matrices with dimension $K^2 \times K^2$. ∎

Remark 10: For the general $M \times N$ X channel with L-antenna relays, we can also design a transmission scheme that first uses partial interference alignment to align L interfering signals, and then uses joint beamforming to align the rest of the interfering signals. The scheme can be designed using similar ideas as in the proof of *Corollary 1* and *Corollary 2*, and thus is omitted here. ∎

C. K-user X channel with J Single Antenna Relays

We now consider the K-user X channel with multiple single-antenna relays. From *Theorem 1*, the condition to achieve the same optimal DoF as the case when CSIT is available is summarized in the following corollary.

Corollary 3: For the K-user X channel with single antenna relays, when there is no CSIT but global CSI is available at the relays, a sufficient condition to achieve the optimal DoF $\frac{K^2}{2K-1}$ is to have $(K-1)^2$ relays.

Remark 11: *Corollary 3* showed that if there are not enough number of antennas at the relays, we can use more relays to compensate the lack of spatial dimensions. If we consider the total number of antennas at all the relays, we can see that the lack of spatial dimensions at the relays increases the total number of antennas needed to achieve the optimal DoF from $K-1$ to $(K-1)^2$. ∎

Remark 12: For K-user X channel with single antenna relays, the number of relays required to achieve the DoF upper bound is $\mathcal{O}(K^2)$. This clearly places more overhead for the relays to obtain the global CSI as compared to obtaining global CSI at the K transmitters. If we want to keep a comparable overhead and employ K relays only, we can only allow $\lfloor \sqrt{K} + 1 \rfloor$ users to transmit, which yields a DoF to the order of $\mathcal{O}(\sqrt{K})$. ∎

We have seen that for the X channel without CSIT, relaying can provide DoF gain to achieve the optimal DoF. It is trivial to see that the same is true for the setting when the transmitters

have delayed CSIT, since one can always ignore the delayed CSIT and employ the same scheme. We next consider the case where the relays have delayed CSI.

D. Full CSI vs Delayed CSI at the Relay

In this section, we investigate the DoF of the K-user X channel without CSIT with one $(K-1)$-antenna relay under the assumption that the relay has delayed CSI. We first consider the K-user X channel with one K-antenna relay, which clearly provides a DoF upperbound to the case with a $(K-1)$-antenna relay.

Theorem 2: For the K-user X channel with a K-antenna relay, when there is no CSIT and only delayed CSI is available at the relay, the DoF is given by

$$\frac{K}{1 + \frac{1}{2} + \cdots + \frac{1}{K}} \tag{52}$$

Proof: The achievability of this DoF can be obtained using a similar strategy as in [12]. The scheme in [12] is designed for the K-user broadcast channel and consists of K phases, where in phase 1, the transmitter sends the messages to the receivers. In slot $t = 1, \cdots, K$ for phase 1, the transmitter sends $\mathbf{X}(t) = (d_{t1}, d_{t2}, \cdots, d_{tK})^T$, where d_{ti} is the ith message intended for receiver t. The transmission scheme used for this phase can be implemented for the K-user X channel. Since the relay has K antennas and delayed CSI, it can decode all the messages, and then it can act as the transmitter in the broadcast channel to implement the transmission scheme for the rest of the phases to achieve the DoF specified by (52).

To upper bound the DoF of the channel, we combine all the transmitters and the relay, which yields a broadcast channel with $2K$ antennas at the transmitter with delayed CSIT. The outerbounds in references [12] and [15] can then be used to obtain equation (52). ∎

Recall that for the K-user X channel without CSIT, when the relay has global CSI, we can achieve the optimal DoF $\frac{K^2}{2K-1}$ with only $K-1$ antennas at the relay. For the case with delayed CSI at the relay, when the relay has $K-1$ antennas, the DoF at most equals equation (52). It is clear that for the K-user X channel without CSIT, global CSI at the relay can provide a DoF gain, compared to the case when only delayed CSI is available at the relay.

IV. RELAY-AIDED INTERFERENCE ALIGNMENT FOR K-USER INTERFERENCE CHANNEL

In this section, we investigate impact of relays on the DoF of the K-user interference channel without CSIT, letting the relays utilize the time/frequency/spatial dimensions available to steer the signals into the desired directions. The goal is once again to recover the optimal DoF with CSIT. Relays are assumed to have global CSI. Following similar arguments as in *Proposition 1*, we first propose a DoF upper bound for this channel.

Proposition 2: The DoF for the K-user interference channel without CSIT but with the presence of relays with global CSI is upper bounded by $\frac{K}{2}$.

Proof: The DoF for the K-user interference channel without CSIT with relays can be upper bounded by the K-user interference channel with CSIT and relays. Since relaying does not provide any DoF gain for interference channel with global CSI at all nodes [30], the optimal DoF for K-user interference channel with full CSIT, which is shown in reference [3] to be $\frac{K}{2}$, can be an upper bound for the K-user interference channel without CSIT with relays. ∎

A. J relays with L antennas

We first consider the most general case for the K-user interference channel, where we have J relays each equipped with L antennas.

Theorem 3: For the relayd- aided K-user interference channel without CSIT, when there is global CSI at the relays, the optimal DoF $\frac{K}{2}$ can be achieved using $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays with L antennas.

Proof: To show the achievability of DoF $\frac{K}{2}$, we construct a 2-slot transmission scheme.

In the first slot, each transmitter sends a message to the intended receiver, i.e.,

$$X_k(1) = d_k, \tag{53}$$

where d_k denotes the data stream carrying the message w_k, and $k = 1, \cdots, K$.

The signals received at receiver k and relay R_j are

$$Y_k(1) = \sum_{i=1}^{K} h_{ki}(1) d_i, \tag{54}$$

$$\mathbf{Y}_{R_j}(1) = \sum_{i=1}^{K} \mathbf{h}_{R_j i}(1) d_i, \tag{55}$$

where $\mathbf{Y}_{R_j}(1), \mathbf{h}_{R_j i}(1) \in \mathbb{C}^L$.

Since we use a 2-slot transmission scheme, the signal space at the receivers has 2 dimensions in time. To decode the intended message, the receivers need to keep all the other $K-1$ interference signals aligned in a one dimensional space. To this end, relay R_j applies a precoding matrix to the received signal vector, and transmits the following signal vector in the second slot:

$$\mathbf{X}_{R_j}(2) = \mathbf{U}_j \mathbf{Y}_{R_j}(1) \tag{56}$$

where $\mathbf{U}_j \in \mathbb{C}^{L \times L}$, which is to be determined later. In the second slot, we also let the receiver perform joint beamforming to transmit

$$X_k(2) = d_k. \tag{57}$$

The received signal at receiver k for slot 2 can be expressed as

$$Y_k(2) = \sum_{i=1}^{K} h_{ki}(2) d_i + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2) \mathbf{x}_{R_j} \tag{58}$$

$$= \sum_{i=1}^{K} h_{ki}(2) d_i + \sum_{j=1}^{J} \sum_{i=1}^{K} \mathbf{h}_{kR_j}^T(2) \mathbf{U}_j \mathbf{h}_{R_j i}(1) d_i. \tag{59}$$

Grouping the received signals at receiver k from 2 slots into vector form, we have

$$\mathbf{Y}_k = \begin{pmatrix} h_{kk}(1) \\ h_{kk}(2) + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_jk}(1) \end{pmatrix} d_k$$
$$+ \sum_{i \neq k} \begin{pmatrix} h_{ki}(1) \\ h_{ki}(2) + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_ji}(1) \end{pmatrix} d_i. \quad (60)$$

In order to align all the interference signals into a one dimensional space, we need

$$\frac{h_{ki}(2) + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_ji}(1)}{h_{ki}(1)}$$
$$= \frac{h_{kl}(2) + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_jl}(1)}{h_{kl}(1)} \quad (61)$$

where $i = 2$ if $k = 1$, and $i = 1$ if $k \neq 1$, for all $l \neq k, l \neq i$. Equation (61) can be equivalently written as

$$\sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j \left(\frac{\mathbf{h}_{R_ji}(1)}{h_{ki}(1)} - \frac{\mathbf{h}_{R_jl}(1)}{h_{kl}(1)} \right)$$
$$= \left(\frac{h_{kl}(2)}{h_{kl}(1)} - \frac{h_{ki}(2)}{h_{ki}(1)} \right) .. \quad (62)$$

If we denote the entries of $\mathbf{U}_j$ as $u_{j,mn}$, where $m, n = 1, \cdots, L$, entries of $\mathbf{h}_{kR_j}(2)$ as $h_{kR_j,m}(2)$, and entries of $\mathbf{h}_{R_ji}(1)$ as $h_{R_ji,n}(1)$, then equation (62) can be written as

$$\sum_{j=1}^{J} \sum_{m=1}^{L} \sum_{n=1}^{L} h_{kR_j,m}(2) \left(\frac{h_{R_ji,n}(1)}{h_{ki}(1)} - \frac{h_{R_jl,n}(1)}{h_{kl}(1)} \right) u_{j,mn}$$
$$= \left(\frac{h_{kl}(2)}{h_{kl}(1)} - \frac{h_{ki}(2)}{h_{ki}(1)} \right). \quad (63)$$

If we let

$$\mathbf{h}_{kl} = \left[h_{kR_j,m}(2) \left(\frac{h_{R_ji,n}(1)}{h_{ki}(1)} - \frac{h_{R_jl,n}(1)}{h_{kl}(1)} \right) \right]^{(jmn)}, \quad (64)$$

$$\mathbf{b} = \left[\frac{h_{kl}(2)}{h_{kl}(1)} - \frac{h_{ki}(2)}{h_{ki}(1)} \right]^{(kl)} \quad (65)$$

and reorganize $u_{j,mn}$ to form a vector

$$\mathbf{u} = [u_{j,mn}]^{(jmn)}, \quad (66)$$

then all the linear equations can be written as

$$\mathbf{Hu} = \mathbf{b}, \quad (67)$$

where $\mathbf{H}$ is obtained by using $\mathbf{h}_{kl}^T$ as its rows for all the enumeration of k and l, corresponding to the order of indices k and l in $\mathbf{b}$.

The matrix $\mathbf{H}$ has dimension $K(K-2) \times JL^2$, and it is full rank almost surely since the entries of channel matrices are drawn from a continuous distribution. In order to guarantee that the interference is aligned, we need to have $JL^2 \geq K(K-2)$ such that we can find precoding matrices $\mathbf{U}_j$ at the relays. When $JL^2 \geq K(K-2)$, matrices $\mathbf{U}_j$ can be obtained from the null space of matrix $\mathbf{H}$ or inverting the matrix $\mathbf{H}$.

Now we need to show that the interference and the signal carrying intended messages are linearly independent. We first observe that when $JL^2 \geq K(K-2)$, $\mathbf{u} = \mathbf{H}^\dagger(\mathbf{HH}^\dagger)^{-1}\mathbf{b}$ is always a solution. The matrices $\mathbf{U}_j$ are thus only linear functions of the channel coefficients except for $h_{kk}(1)$ and $h_{kk}(2)$. From equation (60), since interference is aligned, we have

$$\lambda h_{ki}(1) = h_{ki}(2) + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_ji}(1) \quad (68)$$

for some λ. If the signal carrying intended messages and the interference are also aligned, we must have

$$\lambda h_{kk}(1) = h_{kk}(2) + \sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_jk}(1). \quad (69)$$

Since $\sum_{j=1}^{J} \mathbf{h}_{kR_j}^T(2)\mathbf{U}_j\mathbf{h}_{R_jk}(1)$ is a linear function of channel coefficients except for $h_{kk}(1)$ and $h_{kk}(2)$, the probability that the signal carrying intended messages and the interference are also aligned is zero, since the channel matrices are generated from a continuous distribution. Therefore the receivers can decode the intended messages using zero-forcing, and the DoF $\frac{K}{2}$ can be achieved almost surely. ∎

Remark 13: In the above scheme, when we have $JL^2 = K(K-2)$, the matrix $\mathbf{H}$ in equation (67) is invertible. For this case, we must use joint beamforming and the channel need to be time varying in order to obtain non-zero precoding matrices $\mathbf{U}_j$ at the relays. This is because when we do not use joint beamforming or the channel being constant, the vector $\mathbf{b}$ on right hand side of equation (67) becomes zero, which results in all-zero precoding matrices at the relay. This reduces the available dimensions of the signal space at the receivers to one, similar as the observation we have for the X channel in *Remark 2*. For this case, the intended signal and the interfering signals are aligned together. By remaining silent, the relays are still able to keep all the interference aligned. However, we need another dimension in the signal space to separate the intended signal from the interference. Joint beamforming and time varying channel, for this case, allow the relays to facilitate interference alignment without reducing the dimensions of the signal spaces at the receivers. On the other hand, when we have $JL^2 > K(K-2)$, we can always find a non-zero vector $\mathbf{u}$ from the null space of $\mathbf{H}$, and thus the channel does not need to be time varying and we do not have to use joint beamforming. ∎

Remark 14: In reference [3], the DoF $\frac{K}{2}$ for the K-user interference channel is achieved via channel extension, which requires infinite channel uses to achieve exactly $\frac{K}{2}$ degrees of freedom. In our scheme, however, the DoF is achieved via a two-slot transmission scheme. ∎

Remark 15: If we assume that the channel coefficients are drawn from the Rayleigh distribution, then it is shown in [9] that the DoF for the K-user interference channel without CSIT is upper bounded by 1. It is thus clear that relays can provide DoF gain for the K-user interference channel without CSIT. ∎

Remark 16: The scheme we used for *Theorem 3* can also be applied to the case when the transmitters and the receivers have multiple antennas. ∎

We next consider two special cases of the channel, namely the case when there is a single relay with multiple antennas and the case when there are multiple relays with a single antenna.

B. Single relay with multiple antennas

For this case, it is easy to see that when a relay has K antennas, the DoF upper bound $\frac{K}{2}$ can be achieved using a 2-slot transmission scheme: In the first slot, the transmitters send messages to the relay, and relay decodes all messages. In the second slot, the relay broadcasts all the messages to the receivers. The K antennas at the relay can provide sufficient spatial dimensions for the relay to decode and broadcast the messages. However, from *Theorem 3*, a sufficient condition to achieve the DoF $\frac{K}{2}$ is to have a relay with $K - 1$ antennas, which is summarized in the following corollary:

Corollary 4: For the relay-aided K-user interference channel without CSIT, a sufficient condition to achieve the optimal DoF $\frac{K}{2}$ is to have $K - 1$ antennas at the relay.

This result can be obtained as a special case from *Theorem 3*. Note that this result was also obtained in [39] using similar ideas. In fact, for this case, it is shown in [39] that the $K - 1$ antennas at the relay is also a necessary condition to achieve the optimal DoF using linear precoding schemes at the relay.

From *Remark 13*, we observe two important features for the case with a single relay equipped with $K - 1$ antennas: The channel does not need to be time varying and there is no need for joint beamforming between transmitters and the relay for the transmission in the second slot.

C. Multiple relays with single antenna

We now focus on the case when relays only have a single antenna, and investigate how many relays are needed to achieve the DoF $\frac{K}{2}$. From *Theorem 3*, we have the following corollary.

Corollary 5: For relay-aided K-user interference channel without CSIT, using the presence of single antenna relays with global CSI, a sufficient condition to achieve the optimal DoF $\frac{K}{2}$ is to have $K(K - 2)$ relays.

Different from *Corollary 4*, for the case when we have $K(K - 2)$ relays with single antenna, joint beamforming between the transmitters and the relays and the channel being time varying are two important conditions to achieve the optimal DoF, as observed from *Remark 13*.

Remark 17: The above scheme requires the number of single-antenna relays to be of the order $\mathcal{O}(K^2)$, to achieve the optimal DoF for the K-user interference channel with relays. It is then interesting to see how much DoF we can achieve if the number of relays is of the order $\mathcal{O}(K)$. For this case, we can consider a subset of $\lfloor \sqrt{K} \rfloor$ transmitter-receiver pairs as a $\lfloor \sqrt{K} \rfloor$-user interference channel. The achievable DoF is then $\frac{\lfloor \sqrt{K} \rfloor}{2}$, which is still a significant improvement compared to the DoF of the K-user interference channel with no relays under Rayleigh fading [9]. ∎

Remark 18: For the K-user interference channel with relays, we can also design a two-hop transmission scheme.

However, this requires more relays in general. Reference [36] considered a two-hop interference network with single antenna relays, and showed that to achieve interference-free transmission, which implies achieving DoF $\frac{K}{2}$, we need $K(K-1)+1$ relays. This is more than $K(K-2)$ relays that are needed for our scheme. This is because in our scheme, there are more dimension in the signal space that we can utilize due to the fully connected nature of the channel and the interaction between transmitters and the relays in the transmission in slot 2. ∎

V. CONCLUSION

In this paper, we have investigated relay-aided interference alignment schemes for the X channel and the interference channel, when no channel state information (CSI) at the transmitters (CSIT) is available. In particular, we have considered models where intermediate relay nodes have access to CSI, and can compensate for the lack of CSI at the transmitters. We have first investigated the $M \times N$ X channel without CSIT assisted by relays with global CSI. We have designed a transmission scheme and established sufficient conditions between the number of relays and the number of antennas at the relays such that the same optimal DoF as the case when CSIT is available can be achieved. For the K-user interference channel without CSIT, we have shown that relays can provide interference alignment to achieve the optimal DoF $\frac{K}{2}$ using a 2-slot transmission scheme. In general, we have shown that the optimal DoF $\frac{K}{2}$ can be achieved using $\left\lceil \frac{K(K-2)}{L^2} \right\rceil$ relays with L antennas.

In this paper, the focus has been on recovering the optimal DoF using relays with global CSI, as if transmitters had global CSI when in reality they have none. An interesting direction is quantifying the impact of partial or delayed CSI at the relays on the DoF in the presence of delayed or zero CSI at the transmitters. This is left as future work.

APPENDIX A
PROOF OF THEOREM 2

We denote the message from transmitter i to receiver j as d_{ji}. We wish to send K^2 messages in $2K - 1$ channel uses. In the first K slots, the transmitters send the messages to the relay and the receivers, and in the rest $K - 1$ slots, the relay performs partial interference alignment and joint beamforming with the transmitters to align all the interference into a $K - 1$ dimensional space.

For slot $t = 1, \cdots, K$, transmitter k sends

$$X_k(t) = d_{tk} \tag{74}$$

The signal received at receiver $m \in \{1, 2, \cdots, K\}$ for slot t is

$$Y_m(t) = \sum_{k=1}^{K} h_{mk}(t) d_{tk} \tag{75}$$

$$\mathbf{Y}_R(t) = \sum_{k=1}^{K} \mathbf{h}_{Rk}(t) d_{tk} \tag{76}$$

where $\mathbf{Y}_R(t) \in \mathbb{C}^{K-1}$.

$$Y_m(t') = \sum_{k=1}^{K} h_{mk}(t')d_{kk} + \mathbf{h}_{mR}(t')^T \mathbf{X}_R(t') \tag{70}$$

$$= \sum_{k=1}^{K} h_{mk}(t')d_{kk} + \mathbf{h}_{mR}(t')^T \sum_{t \neq m, i=m} \alpha_{tm}(t')\mathbf{v}_{tm}(t')\left(\mathbf{u}_m(t)^T\mathbf{h}_{Rt}(t)d_{tt} + \sum_{n \neq t} h_{mn}(t)d_{tn}\right) + \mathbf{h}_{mR}(t')^T \cdot$$

$$\sum_{t=m, i \neq m} \alpha_{mi}(t')\mathbf{v}_{mi}(t')\left(\mathbf{u}_i(m)^T\mathbf{h}_{Rm}(m)d_{mm} + \sum_{n \neq m} h_{in}(m)d_{mn}\right) \tag{71}$$

$$
\mathbf{Y}_m = \begin{matrix} m \\ t' \end{matrix} \begin{bmatrix} \mathbf{0} \\ h_{mm}(m) \\ \mathbf{0} \\ \left[h_{mm}(t') + \sum_{i \neq m} \alpha_{mi}(t')\mathbf{h}_{mR}(t')^T\mathbf{v}_{mi}(t')\mathbf{u}_i(m)^T\mathbf{h}_{Rm}(m)\right]^{(t')} \end{bmatrix} d_{mm}
$$
$$
+ \sum_{k \neq m} \begin{matrix} m \\ t' \end{matrix} \begin{bmatrix} \mathbf{0} \\ h_{mk}(m) \\ \mathbf{0} \\ \left[\sum_{i \neq m} \alpha_{mi}(t')\mathbf{h}_{mR}(t')^T\mathbf{v}_{mi}(t')h_{ik}(m)\right]^{(t')} \end{bmatrix} d_{mk}
$$
$$
+ \sum_{\gamma \neq m} \left(\begin{matrix} \gamma \\ t' \end{matrix} \begin{bmatrix} \mathbf{0} \\ h_{m\gamma}(\gamma) \\ \mathbf{0} \\ \left[h_{m\gamma}(t') + \alpha_{\gamma m}(t')\mathbf{h}_{mR}(t')^T\mathbf{v}_{\gamma m}(t')\mathbf{u}_m(\gamma)^T\mathbf{h}_{R\gamma}(\gamma)\right]^{(t')} \end{bmatrix} d_{\gamma\gamma} \right.
$$
$$
\left. + \sum_{k \neq \gamma} \begin{matrix} \gamma \\ t' \end{matrix} \begin{bmatrix} \mathbf{0} \\ h_{mk}(\gamma) \\ \mathbf{0} \\ \left[\alpha_{\gamma m}(t')\mathbf{h}_{mR}(t')^T\mathbf{v}_{\gamma m}(t')h_{mk}(\gamma)\right]^{(t')} \end{bmatrix} d_{\gamma k} \right) \tag{72}
$$

$$\alpha_{\gamma m}(t') = \frac{h_{m\gamma}(t')}{h_{m\gamma}(\gamma)\mathbf{h}_{mR}(t')^T\mathbf{v}_{\gamma m}(t')h_{mk}(\gamma) - \mathbf{h}_{mR}(t')^T\mathbf{v}_{\gamma m}(t')\mathbf{u}_m(\gamma)^T\mathbf{h}_{R\gamma}(\gamma)} \tag{73}$$

Now we need to obtain the vectors $\mathbf{u}_i(t) \in \mathbb{C}^{K-1}$ to partially align the interference at the receivers. We let

$$\mathbf{u}_i(t)^T\mathbf{h}_{Rk}(t) = h_{ik}(t) \tag{77}$$

where $k \neq t, i \neq t$. Since we have exactly $K - 1$ equations to solve for $K - 1$ variables, and the channel coefficients are drawn from the continuous distribution, there exist non-zero vectors $\mathbf{u}_i(t)$ almost surely.

We then have

$$\mathbf{X}_R^i(t) = \mathbf{u}_i(t)^T\mathbf{Y}_R(t) = \mathbf{u}_i(t)^T\mathbf{h}_{Rt}(t)d_{tt} + \sum_{k \neq t} h_{ik}(t). \tag{78}$$

For each $X_R^i(t)$, we choose a weighting coefficient $\alpha_{ti}(t')$, where $t' = K + 1, \cdots, 2K - 1$, and a beamforming vector $\mathbf{v}_{ti}(t')$. We choose the beamforming vectors such that $\mathbf{v}_{ti}(t') \in \mathcal{N}([\{\mathbf{h}_{lR}(t')\}]^T)$, where $[\{\mathbf{h}_{lR}(t')\}]$ denotes a matrix taking the vector $\mathbf{h}_{lR}(t')$ as its columns for all $l \neq t, l \neq i$. The matrix $[\{\mathbf{h}_{lR}(t')\}]^T$ has dimension $(K - 2) \times (K - 1)$, and thus its null space is non-empty, which guarantees the existence of $\mathbf{v}_{ti}(t')$.

For slots $t' = K+1, K+2, \cdots, 2K-1$, the relay transmits

$$\mathbf{X}_R(t') = \sum_{t=1}^{K} \sum_{i \neq t} \alpha_{ti}(t')\mathbf{v}_{ti}(t')\mathbf{u}_i(t)^T\mathbf{Y}_R(t). \tag{79}$$

In the meantime, the transmitters send

$$X_k(t') = d_{kk}. \tag{80}$$

The received signal at receiver m for slot t' is $Y_m(t')$ as shown in equation (71) at the beginning of this page.

We then combine the received signals from $2K - 1$ slots into one vector $\mathbf{Y}_m$ as shown in equation (72) at the beginning of this page.

For receiver m, d_{mk}, $k = 1, \cdots, K$, are the messages that it needs to decode, which should span a K dimensional space. There are a total of $2K - 1$ dimensions available for the received signals, and hence we should align the rest interference signals into a $K - 1$ dimensional space. With the help of the relay, we have already aligned the interfering data streams $d_{\gamma k}, \forall k \neq \gamma$, into a one dimensional space for each fixed γ. If we can steer the data stream $d_{\gamma\gamma}$ into the same dimension of the signal space, then we are able to keep all the interference into a $K - 1$ dimensional space.

This is feasible by choosing the parameters $\alpha_{\gamma m}(t')$ such that equation (73) at the beginning of previous page is satisfied for all $t' = K+1, \cdots, 2K-1$.

It is easy to verify that after aligning the interference, the intended messages d_{mk} occupy a K-dimensional space, which does not intersect with the $(K-1)$-dimensional space of the interfering signals, and thus they can be decoded using a zero-forcing decoder to completely eliminate the interference. Therefore we are able to send K^2 messages with $2K-1$ slots, and the DoF $\frac{K^2}{2K-1}$ is achievable.

REFERENCES

[1] S. A. Jafar and S. Shamai, "Degrees of freedom region for the MIMO X channel," *IEEE Transactions on Information Theory*, vol. 54, no. 1, pp. 151–170, January 2008.

[2] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom of wireless X networks," *IEEE Transactions on Information Theory*, vol. 55, no. 9, pp. 3893–3908, September 2009.

[3] ——, "Interference alignment and the degrees of freedom for the K-user interference channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, August 2008.

[4] V. Cadambe, S. A. Jafar, C. Wang, "Interference Alignment with Asymmetric Complex Signaling – Settling the Host-Madsen-Nosratinia Conjecture", *IEEE Transactions on Information Theory*, vol. 56, no, 9, pp. 4552–4565, September 2010.

[5] T. Gou and S. A. Jafar, "Degrees of freedom of the K user MxN MIMO interference channel," *IEEE Transactions on Information Theory*, vol. 56, no. 12, pp. 6040–6057, December 2010.

[6] ——, "Capacity of a class of symmetric SIMO Gaussian interference channels within O(1)," *IEEE Transactions on Information Theory*, vol. 57, no. 6, pp. 1932–1958, April 2011.

[7] C. Huang and S. A. Jafar, "Degrees of freedom of the MIMO interference channel with cooperation and cognition," *IEEE Transactions on Information Theory*, vol. 55, no. 9, pp. 4211–4220, September 2009.

[8] C. Huang, S. A. Jafar, S. Shamai, and S. Vshwanath, "On degrees of freedom region of MIMO networks without channel state information at transmitters," *IEEE Transactions on Information Theory*, vol. 58, no. 2, pp. 849–857, February 2012.

[9] C. S. Vaze and M. Varanasi, "The degrees of freedom regions of MIMO broadcast, interference, and cognitive radio channels with no CSIT," *IEEE Transactions on Information Theory*, vol. 58, no. 8, pp. 5354–5374, August 2012.

[10] S. A. Jafar, "Blind Interference Alignment," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 9, pp. 216–227, June 2012.

[11] C. Wang, T. Gou, and S. A. Jafar, "Aiming perfectly in the dark - blind interference alignment through staggered antenna switching," *IEEE Transactions on Signal Processing*, vol. 59, no. 6, pp. 2734–2744, June 2011.

[12] M. A. Maddah-Ali and D. Tse, "Completely stale transmitter channel state information is still very useful," *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4418–4431, July 2012. September 2010.

[13] H. Maleki, S. A. Jafar, and S. Shamai, "Retrospective interference alignment over interference networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 3, pp. 228–240, June 2012.

[14] C. S. Vaze and M. Varanasi, "The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSI," *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4396–4417, July 2012, accepted, available at arXiv:1101.5809v2 [cs.IT].

[15] C. S. Vaze and M. K. Varanasi, "The degrees of freedom region of the two-user MIMO broadcast channel with delayed CSIT," in *Proceedings of IEEE International Symposium on Information Theory*, July 2011.

[16] A. Ghasemi, A. S. Motahari, and A. K. Khandani, "Interference alignment for the MIMO interference channel with delayed local CSIT," *submitted to IEEE Transactions on Information Theory*, available at http://arxiv.org/abs/1101.5809.

[17] ——, "On the degrees of freedom of X channel with delayed CSIT," in *Proceedings of IEEE International Symposium on Information Theory*, July 2011.

[18] R. Tandon, S. Mohajer, H. V. Poor, and S. Shamai, "Degrees of freedom region of the MIMO interference channel with output feedback and delayed CSIT," *IEEE Transactions on Information Theory*, vol. 59, no. 3, pp. 1444–1457, March 2013.

[19] M. J. Abdoli, A. Ghasemi, and A. K. Khandani, "On the degrees of freedom of three-user MIMO broadcast channel with delayed CSIT," in *Proceedings of IEEE International Symposium on Information Theory*, July 2011.

[20] C. Shannon, "The zero error capacity of a noisy channel," *IEEE Transactions on Information Theory*, vol. 2, no. 3, pp. 8–19, September 1956.

[21] H. Viswanathan, "Capacity of markov channels with receiver CSI and delayed feedback," *IEEE Transactions on Information Theory*, vol. 45, no. 2, pp. 761–771, March 1999.

[22] U. Basher, A. Shirazi, and H. Permuter, "Capacity region of finite state multiple-access channel with delayed state information at the transmitters," *IEEE Transactions on Information Theory*, vol. 58, no. 6, pp. 3430–3452, June 2012.

[23] T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. IT-25, pp. 572–584, 1979.

[24] L. Sankaranarayanan, G. Kramer, and N. B. Mandayam, "Offset encoding for multiple-access relay channels," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3814–3821, October 2007.

[25] R. Tandon and H. V. Poor, "On the capacity region of multiple-access relay channels," in *Proceedings of Conference on Information Sciences and Systems, Baltimore, MD*, March 2011.

[26] Y. Tian and A. Yener, "The Gaussian interference relay channel: Improved achievable rates and sum rate upperbounds using a potent relay," *IEEE Transactions on Information Theory, Special Issue on Interference Networks*, vol. 57, no. 5, pp. 2865–2879, May 2011.

[27] ——, "Symmetric capacity of the Gaussian interference channel with an out-of-band relay to within 1.15 bits," *IEEE Transactions on Information Theory*, vol. 58, no. 8, pp. 5151–5171, August 2012.

[28] O. Sahin, O. Simeone, and E. Erkip, "Interference channel with an out-of-band relay," *IEEE Transactions on Information Theory, Special Issue on Interference Networks*, vol. 57, no. 5, pp. 2746–2764, May 2011.

[29] I. Maric, R. Dabora, and A. Goldsmith, "Generalized relaying in the presence of interference," in *Proceedings of the 42nd Asilomar Conference on Signals, Systems and Computers*, October 2008.

[30] V. R. Cadambe and S. A. Jafar, "Degrees of freedom of wireless networks with relays, feedback, cooperation and full duplex operation," *IEEE Transactions on Information Theory*, vol. 55, no. 5, pp. 2334–2344, May 2009.

[31] B. Nourani, S. A. Motahari, and A. K. Khandani, "Relay-aided interference alignment for the quasi-static X channel," in *Proceedings of IEEE International Symposium on Information Theory*, June 2009.

[32] ——, "Relay-aided interference alignment for the quasi-static interference channel," in *Proceedings of IEEE International Symposium on Information Theory*, June 2010.

[33] D. Jin, J. No, and D. Shin, "Interference alignment aided by relays for the quasi-static X channel," in *Proceedings of IEEE International Symposium on Information Theory*, July 2011.

[34] H. Ning, C. Ling, and K. K. Leung, "Relay-aided interference alignment: Feasibility conditions and algorithm," in *Proceedings of IEEE International Symposium on Information Theory*, June 2010.

[35] R. Tannious and A. Nosratinia, "Relay-assisted interference network: Degrees of freedom," *IEEE Transactions on Information Theory*, vol. 58, no. 6, pp. 3780–3786, June 2012.

[36] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, February 2007.

[37] S.-W. Jeon, S.-Y. Chung, and S. A. Jafar, "Degrees of freedom region of a class of multisource Gaussian relay networks," *IEEE Transactions on Information Theory*, vol. 57, no. 5, pp. 3032–3044, May 2011.

[38] T. Gou, S. A. Jafar, C. Wang, S.-W. Jeon, and S.-Y. Chung, "Aligned interference neutralization and the degrees of freedom of the 2x2x2 interference channel," *IEEE Transactions on Information Theory*, vol. 58, no. 7, pp. 4381–4395, July 2012.

[39] S. Chen and R. S. Cheng, "Achieve the degrees of freedom of K-user MIMO interference channel with a MIMO relay," in *Proceedings of the IEEE Global Telecommunications Conference*, December 2010.